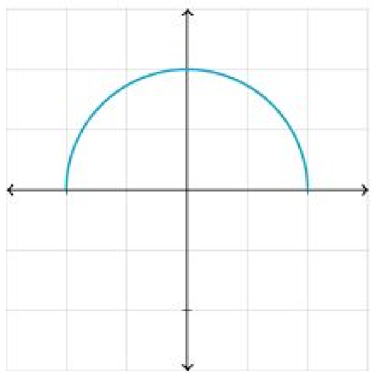
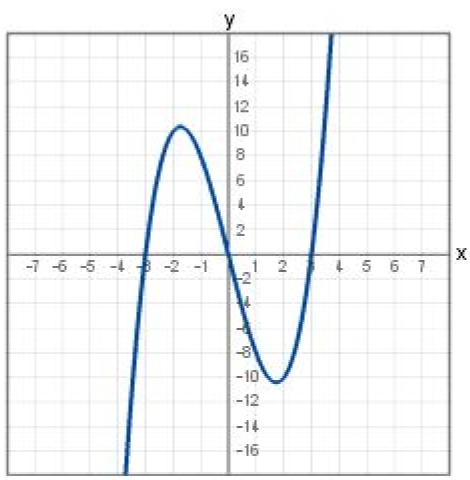
Maxes/Mins of Graphs

- Simply put, maxes and mins of graphs are the highest and lowest points of a graph or a section of graph
- Maxes/mins of a section of graph can only occur either when the slope of the graph is changing either from negative to positive, positive to negative, or at one of the ends of the section
- Both can occur at either endpoint, but otherwise maxes only occur when slope goes from positive to negative, and mins vice versa negative to positive

Visuals:



As you can with this graph y=√(1-x^2), the highest point occurs at (0,2), and the lowest points occur at the ends at (-2,0) and (2,0). Since there is only one real max on this graph: (0,2) is both the relative and absolute max for the graph. There are two mins on the graph, at the same exact level, so they are both absolute mins.



With this next graph you can just look again visually to find the mins and maxes. In cases with many maxes/mins sometimes it it easiest to go left to right on the graph so as not to overlook any. The first point to look at is the left endpoint, which is clearly a min at around (-3.6, -18). The next important points to look for is when the slope changes sign. At around (-1.8, 10.4) there is a max. The slop goes negative until around (1.8,-10.4) where there is a min. Finally there is the right endpoint (3.6,18) which is another max. There is only one absolute max ever on a graph, the highest point. In this case it is (3.6,18). There is also only one absolute min, which is the lowest point, at (-3.6,-18). (1.8,-10.4) isn't the absolute min, therefore it is only a relative min. The same is true of the relative max (-1.8, 10.4).

Finding maxes/mins w/o a graph

- If you aren't given a graph but just an equation and asked to find the maxes or mins, you must use derivatives to find them.
- Remember back to the fact that with the exception of endpoints maxes/mins can only happen when the slope has a sign change. These points are points with a slope of 0, in other words at that point the derivative of the equation equals 0.
- You then need to test points to either side of this to find the slope on either side (plug close pont into the derivative equation).
- If the slope to the left is positive, and to the right it is negative, the point is a max, if the slope to the left is negative and to the right is positive, it is a min (sometimes slope will go to zero but not change sign but this is uncommon)

Ex. [-3,1] of x^2, find the mins and maxes.

1) First find the derivative and points w/ slope of 0.
    a) $d(x^2)/dx= 2x$ $2x=0$, slope is 0 @ $x=0$
2) Now you have your important values: the endpoints and $x=0$
    a) These are $x=-3$ $x=0$ and $x=1$
3) Next test the slope between theo see which way the sign changes, do this by picking a number and plugging it into the derivative equation
    a) First, between $x=-3$ and $x=0$, I will pick $x=-1$ and plug in: $d/dx=2x$, $x=-1$, $d/dx=(2)(-1)=-2$, the slope is negative
    b) Now between $x=0$ and $x=1$, I pick $x=1/2$: $d/dx=2x$ $x=1/2$ $d/dx=(2)(1/2)=1$
4) So now piece it together, the slope is negative from $x=-3$ to $x=0$, and positive from $x=0$ to $x=1$. Since the slope is negative directly to the left of $x=0$, and positive directly to the right, it is a min. Since the slope is negative to the right of

x=-3, and there is nothing to the left, it is a max. Since the slope is positive to the left of x=1, and there is nothing to the right, it is a max also.

5) Plug in normal equation to find absolute vs. relative maxes/mins
    a) x=-3 $y=x^2$ y=9
    b) x=0 $y=x^2$ y=0
    c) x=1 $y=x^2$ y=1
6) The only min is x=0, so it is an absolute min. x=-3 is higher than x=1, so it is the absolute max, while x=1 is a relative max